Chapter 3 SURVEY

3.1 Skeleton Extraction

O'Brien, et al., in 2000 [84] and with Kirk in 2005 [60] have produced a fast method for skeleton formation using a linear least squares method assuming there is a relatively stationary point between two segments, and then solving for that point, which is the rotation point.

3.2 Sphere Estimates

Leendert de Witte [116], in 1960, found a solution for a circle in 3-D space. He used spherical trigonometry to solve the minimized distance from the best great circle path. He used approximations that are convenient for large radii and had to choose from three solutions to get the correct one.

The Maximum Likelihood Estimator (MLE) was the first solution for finding the sphere parameters. In 1961 Stephen Robinson [95] presented the iterative method of solving the sphere by minimizing

$$\varepsilon^2 = \sum_{i=1}^{N}\left(\sqrt{(x_i-\hat{c})^T(x_i-\hat{c})}-\hat{r}\right)^2 \tag{1}$$

There is a closed form solution for the radius estimator but not for the center estimator. Robinson showed the radius estimator to be

$$\hat{r} = \tfrac{1}{N}\sum_{i=1}^{N}\sqrt{(x_i-\hat{c})^T(x_i-\hat{c})} \tag{2}$$

A truly closed form solution wasn't found until another function to minimize was recognized. The new function to minimize was

$$\varepsilon^2 = \sum_{i=1}^{N} \left((x_i - \hat{c})^T (x_i - \hat{c}) - \hat{r}^2 \right)^2 \tag{3}$$

Paul Delogne pioneered what would eventually become the Generalized Delogne-Kása estimator. In 1972, Delogne presented [41] a method for solving a circle for the purposes of determining reflection measurements on transmission lines. Delogne's solution to the circle involves the inverse of a 3x3 matrix. He presented the matrix solution and a rough error analysis as

$$\begin{pmatrix} \hat{c} \\ \hat{c}^T \hat{c} - \hat{r}^2 \end{pmatrix} = \begin{pmatrix} 8 \sum_{i=1}^{N} x_i x_i^T & -4N\overline{x} \\ -4N\overline{x}^T & 2N \end{pmatrix}^{-1} \begin{pmatrix} 4 \sum_{i=1}^{N} x_i x_i^T x_i \\ -2 \sum_{i=1}^{N} x_i^T x_i \end{pmatrix} \tag{4}$$

$$\sigma_i^2 = 2\hat{r} \left| \sqrt{(x_i - \hat{c})^T (x_i - \hat{c})} - \hat{r} \right|. \tag{5}$$

István Kása did more analysis in 1976 [57]. He was the first to recognize the bias in the answer and produced better error analysis.

$$\begin{pmatrix} \hat{c} \\ \hat{r}^2 - \hat{c}^T \hat{c} \end{pmatrix} = \begin{pmatrix} 2N\overline{x}^T & N \\ 2 \sum_{i=1}^{N} x_i x_i^T & N\overline{x} \end{pmatrix}^{-1} \begin{pmatrix} \sum_{i=1}^{N} x_i^T x_i \\ \sum_{i=1}^{N} x_i x_i^T x_i \end{pmatrix} \tag{6}$$

$$\sigma^2 = \frac{1}{4\hat{r}^2 N} \sum_{i=1}^{N} \left((x_i - \hat{c})^T (x_i - \hat{c}) - \hat{r}^2 \right)^2 \tag{7}$$

Vaughan Pratt [93] in 1987 produced a very generic linear least squares method for algebraic surfaces. His solution was slower for spheres due to the need to extract the eigenvectors.

Gander, et. al. in 1994 [47] produced the linear least-squares method for circle fitting with the equation

(8) $$0 = \begin{pmatrix} x_1^T x_1 & x_1^T & 1 \\ \vdots & \vdots & \vdots \\ x_N^T x_N & x_N^T & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2\hat{c} \\ \hat{c}^T \hat{c} - \hat{r}^2 \end{pmatrix}$$

They solved this through finding the null vector of the left matrix that is in essence the singular value decomposition.

Samuel Thomas and Y. Chan in 1995 [108] created a formula for the Cramér-Rao Lower Bound for the circle estimation.

In 1997, Lukács, et. al. [73], produced some improvements on non-linear minimization for spheres.

Corral, et. al. [40] in 1998 analyzed Kása's formula in more detail and a way to reject the answer if the confinement angle got too small.

A paper in nuclear physics describes a method in 2000 to produce the circular arc of a particular traveling in a cloud chamber. Strandlie, et. al. [106] transformed a Riemann sphere into a plane and fit the plane using standard methods involving the eigenvalues of the sample covariance matrix.

Zelniker in 2003 [120] reformulated the circle equation to solve directly for the center using the pseudo-inverse ($^{\#}$) of a 2xN matrix.

(9)
$$\hat{c} = \frac{1}{2}\begin{pmatrix} x_1^T - \overline{x}^T \\ x_2^T - \overline{x}^T \\ \vdots \\ x_N^T - \overline{x}^T \end{pmatrix}^{\#} \begin{pmatrix} x_1^T x_1 - \frac{1}{N}\sum_{i=1}^{N} x_i^T x_i \\ x_2^T x_2 - \frac{1}{N}\sum_{i=1}^{N} x_i^T x_i \\ \vdots \\ x_N^T x_N - \frac{1}{N}\sum_{i=1}^{N} x_i^T x_i \end{pmatrix}$$

Zelniker also came up with a better evaluation of the Cramér-Rao Lower Bound for the center estimator.

(10)
$$Cov\left(\hat{c},\hat{c}^T\right) = r^2\sigma^2 \left(\begin{pmatrix} \mu_1^T - \overline{\mu}^T \\ \mu_2^T - \overline{\mu}^T \\ \vdots \\ \mu_N^T - \overline{\mu}^T \end{pmatrix}^T \begin{pmatrix} \mu_1^T - \overline{\mu}^T \\ \mu_2^T - \overline{\mu}^T \\ \vdots \\ \mu_N^T - \overline{\mu}^T \end{pmatrix} \right)^{-1}$$

Michael Burr, et. al. [31] in 2004 created a geometric inversion technique which far surpasses the complexity needed to solve for a hypersphere. It was a non-linear approach and they failed to recognize there were simpler linear solutions to what they were solving.

In 2007, Knight et al. [63], published the initial results from the research for this dissertation in which a skeleton was formed from a closed-form solution of generic motion capture data.

3.3 Inverse Kinematics

Badler [12] in 1986 produced an interactive 6 DOF controlled technique to produce a skeleton using inverse kinematics and a joystick. Wiley et al. [115] in 1997, came up with a way to splice various regimes of motion capture and skeleton formation based on inverse kinematics. Bodenheim et al. [21], in 1997, produced an articulated skeleton by painstaking hand measurements of the markers and inverse kinematics to optimize the joint angles.

3.4 Kinetics

In 1998 Znamenáček [118] created an efficient recursive algorithm for multibody forward kinetics.