# EXAM 1: Introduction to Computer Graphics CS480/580 

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## Your NAME:

Do all FIVE questions. If you have any questions, please raise your hand and I will get to you. If you need more space to write the answer then use the back of the page. Note that 2-D/3-D transformation matrices are supplied with the exam.

Question 1 (a): (10 points) (2D rotation and scaling) Prove that 2D rotation and scaling commute only if $s_{x}=s_{y}$. Note: $s_{x}$ and $s_{y}$ are the scaling along x and y axis.

Question 1 (b): (5 points) (3D) Direction cosines of a vector are (a, b, c). Shear this vector to ( $0,0, c$ ). Give the Shear matrix and prove your answer.

Question 2: (15 points) (2D Clipping) Using parametric equations, find the intersection point of two lines. The first line starts at $(-2,-2)$ and ends at $(2,2)$. The second line is a vertical line starting at $(0,0)$ and ending at $(0$, 5).

Question 3: (20 points) (2D Scaling in arbitrary direction) Consider a polygon ABCD as shown in the figure below. Scale this polygon by a factor of 2 along the direction from point A at $(0,0)$ to point D at $(2,2)$. Derive the transformation matrix. Calculate the position of all the four points after the transformation. Show that they move in the desired direction. Would the position of point A at $(0,0)$ also change? Why or why not? Explain your answer.
Note: $\cos 45=\sin 45=\frac{1}{\sqrt{2}}$.


Question 4: (20 points) (3D rotations) Direction cosines of two vectors are: $(\mathrm{a}, \mathrm{b}, \mathrm{c})$ and ( $\mathrm{d}, \mathrm{e}, \mathrm{f})$. Use this information to find three orthogonal vectors (defining a right-handed coordinate system) so that one of them is parallel to the vector (d, e, f). Next, give a rotational matrix which will transform these three vectors so that ( $\mathrm{d}, \mathrm{e}, \mathrm{f}$ ) becomes parallel to the x -axis after the rotation.

Question 5 (a): (10 points) What is the main advantage of using homogeneous coordinate system for manipulating rotation, translation and scaling matrices? How would you represent a point at infinity in homogeneous coordinate system?

Question 5 (b): (20 points) (3D) A rectangular box is defined by $\mathrm{a} \leq \mathrm{x}$ $\leq \mathrm{d} ; \mathrm{b} \leq \mathrm{y} \leq \mathrm{e} ; \mathrm{c} \leq \mathrm{z} \leq \mathrm{f}$. Transform so that the box becomes: $2 \leq \mathrm{x} \leq 4$; $1 \leq \mathrm{y} \leq 4 ;-1 \leq \mathrm{z} \leq 2$. Explain your transformation.

