Incentives for Crypto-Collateralized Digital Assets

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Abstract

Digital currencies such as Bitcoin frequently suffer from high price volatility, limiting their utility as a means of purchasing power. Hence, a popular topic among cryptocurrency researchers is a digital currency design which inherits the decentralization of Bitcoin while somehow mitigating its violent price swings. One such system which attempts to establish a price-stable cryptocurrency is the BitShares market-pegged-asset protocol. In this paper, we present a simple mathematical model of the BitShares protocol, and analyze it theoretically and numerically for incentive effects. In particular, we investigate how the selection of two key design parameters function as incentive mechanisms to encourage token holders to commit their core BitShares tokens as collateral for the creation of new price-stabilized tokens. We show a pair of analytical result characterizing some simple facts regarding the interplay between these design parameters. Furthermore, we demonstrate numerically that in some settings, setting these design parameters is a complex, sensitive, and unintuitive task, prompting further work to more fully understand this design process.

I. Introduction

Since the introduction of the Bitcoin cryptocurrency in 2008 [1], digital currencies based on blockchain technology have sprung into the public view in large part due to the dramatic price swings they experience [2]. For example, in the two years from January 1, 2017 to January 1, 2019, the US-Dollar denominated price of the Bitshares core token (BTS) gained approximately 7500%, then lost about 85%, then gained 1600%, and then lost another 95% [3]. It is widely believed that this extreme volatility hinders the consumer adoption of cryptocurrencies for ordinary payments, since the purchasing power of an ordinary currency is expected to change very slowly, if at all [4].

Various methods have been proposed over the past five years to create digital currencies which maintain the decentralized and trustless nature of ordinary cryptocurrencies without suffering from the same price volatility [5], [6]. To date, the longest-running such project is the Bitshares market-pegged-asset protocol (MPA), which has been nearly continuously operational in some form since mid-2014 [7]. The MPA system functions in a largely decentralized manner by allowing core token (BTS) holders to lock away their BTS tokens as collateral in exchange for a loan of bitAsset tokens, which can then be sold on the open market. The protocol has several mechanisms to regulate token supply and collateral ratios for the purpose of maintaining the market price of the pegged assets at or near a price target.

While largely successful in maintaining a price peg (and considerably moreso than some early competitors), two major problems have plagued the MPA system in recent years:

- **Loose price-pegging.** From 2014 to November 2018, the US Dollar-pegged smartcoin BitUSD has maintained an average price *near* \$1, but for extended periods has traded above \$1.15, with occasional drops to around \$0.90. This is a considerably narrower range than typical freely-floating cryptocurrencies, but stands as an important area for improvement.

- **Undercollateralization.** To insulate collateral holders from each others' risky behavior, the MPA system has an in-built safety mechanism known as "global settlement." In the event of undercollateralization, the global settlement mechanism immediately closes all collateral positions and establishes a fixed exchange rate between BTS and the bitAsset, effectively ceasing any form of price-pegging. This mechanism was triggered in late 2018 on several BitShares-platform smartcoins, including BitUSD (which at the time of writing has a market capitalization of approximately $10 million).

In this paper, we present a highly-simplified model of the BitShares price-pegging mechanism, and use it to perform an initial numerical study on some of the key design tradeoffs facing a smartcoin protocol designer. In particular, we study how the selection of two protocol incentive parameters impact the likelihood of undercollateralization events for the price-pegged smartcoins. Our central contributions is to show that the incentive model in BitShares contains regions of high sensitivity; that is, token holders' decisions are discontinuous in incentive parameters in such a way that a small change in parameter values can cause a large sudden jump in overall system risk levels.

II. Model and Performance Metrics

A. Collateral Incentive Model

Due to the complexity of modeling the full problem, in this paper we allow (with loss of generality) the collateralization of a smartcoin to be contolled by a single centralized entity; we term this entity the *agent*. This simplification allows us to closely examine some of the key incentive issues involved in collateral management without any of the complications arising from a multi-agent setting. Throughout, the reader may refer to Table I for a compact depiction of the agent's perspective of the protocol.

The agent is modeled as possessing an endowment of Q core tokens to be committed as collateral. The agent must decide what *value* of stable tokens to be created using the endowment Q as collateral. In other words, the agent must select a *collateral ratio* R, where

$$\langle \text{Value of stable tokens created} \rangle = \frac{Q}{R}. \tag{1}$$

The agent's choice of R is lower-bounded by the blockchain *maintenance collateral ratio* parameter $M \geq 1$; that is,

$$R \geq M.^{1} \tag{2}$$

Once the collateral is committed and stable tokens created, it is assumed that the agent trades the stable tokens on the open market for their equivalent value in core tokens, resulting in the following situation (in all of the following, the amounts are denominated in terms of their equivalent market value of core tokens):

1) Core token collateral held by blockchain: Q.
2) Stable-token debt: Q/R.
3) Core tokens held freely: Q/R.

We consider the effect of a sudden shock $d > 0$ on the underlying core token price; this shock is drawn from some distribution D. Here, we assume for simplicity that the shock occurs, and

[1]In the BitShares blockchain, a typical value of M at press time is 1.5 or 1.6.

then the agent's collateralized position is immediately closed and all profits or losses are realized immediately. According to the BitShares protocol, if the price shock causes the agent's collateral ratio to fall below 1, then all of the agent's collateral is taken to cover the debt. This scenario is known as *global settlement*.

If the shock negative but less severe and the collateral ratio remains above 1 but falls below M, then the agent's debt is multiplied by a penalty factor known as the *maximum short-squeeze ratio* $S \geq 1$; this is known as a *margin call*.

Finally, if the shock is such that the agent's collateral ratio remains above M, no penalty is assessed and in our model the agent simply realizes the resulting loss or gain.

Mathematically, this penalty-free scenario results in a shock-adjusted debt of $\frac{Q}{dR}$, so the agent's profit ratio $P(R, d; M, S)$ is

$$P(R,d;M,S) = \frac{Q/R - Q/(Rd)}{Q} \tag{3}$$

$$= \frac{1}{R}\left(1 - \frac{1}{d}\right). \tag{4}$$

For moderate negative shocks, the agent must pay the additional S penalty for the margin call, resulting in a profit ratio of

$$P(R,d;M,S) = \frac{Q/R - QS/(Rd)}{Q} \tag{5}$$

$$= \frac{1}{R}\left(1 - \frac{S}{d}\right). \tag{6}$$

Finally, for shocks that result in undercollateralization and global settlement, the agent must surrender the entire collateral amount to cover the debt, resulting in

$$P(R,d;M,S) = \frac{Q/R - Q}{Q} \tag{7}$$

$$= \frac{1}{R} - 1. \tag{8}$$

Thus, the full *ex post* profit ratio (that is, the profit given d) is

$$P(R,d;M,S) = \begin{cases} \frac{1}{R} - 1 & \text{if } d \in [0, S/R) \\ \frac{1}{R}\left(1 - \frac{S}{d}\right) & \text{if } d \in [S/R, M/R) \\ \frac{1}{R}\left(1 - \frac{1}{d}\right) & \text{if } d \in [M/R, \infty). \end{cases} \tag{9}$$

When the dependence of $P(R, d; M, S)$ on M and S is clear, we shall sometimes simply write $P(R, d)$.

Table I: Tabular depiction of collateralization incentive model. In the first row, labeled *Start*, the agent holds Q core blockchain tokens, no debt, and no collateral. In stage 2, the agent has committed Q tokens as collateral and received a loan of Q/R price-stable tokens, which are then sold. In stage 3, the value of the core token has changed by a factor of d; since our accounting is being done in core tokens, the effect of this is that the agent's debt is modified by a factor of $1/d$. In stage 4, the debt position is closed, resulting in either a profit or a loss for the agent depending on the M and S parameters.

Stage	**Debt to Blockchain**	**Locked Collateral**	**Freely-held tokens**
Start	0	0	Q
Position Opened	$\frac{Q}{R}$	Q	$\frac{Q}{R}$
After Price Shock	$\frac{Q}{Rd}$	Q	$\frac{Q}{R}$
Position Closed	0	0	$P(R,d;M,S)$ Depends on d; see (9)

B. Decision Model

Let the price shock d be drawn from some distribution D with probability density function $f : [0,\infty) \to [0,\infty)$. Given this distribution, M, and S, the agent's goal is simple: select collateral ratio R to maximize expected profit. Throughout, we use the notation $P(R;M,S)$ to denote the agent's expected profit given distribution D. That is, the agent's optimal collateral ratio is

$$R^*(M,S) \triangleq \operatorname*{arg\,max}_{R\geq M} \mathop{\mathbb{E}}_{d\sim D}\left[P(R,d;M,S)\right]. \tag{10}$$

However, the system designer's goal in selecting incentive parameters M and S is somewhat more nuanced. On the one hand, the designer wishes to ensure that the agent selects a low-enough collateral ratio R so that a reasonable number of price-stable tokens are created. On the other hand, the designer wishes to ensure that the agent selects a *high*-enough R so that the price-stable token remains fully collateralized in the event of a significant market downturn (i.e., the agent's collateral is sufficient to repay the debt even when a very low value of d is realized).

In this paper, we focus simply on the effect of M and S on the probability of an undercollateralization event, and leave the study of the above nontrivial tradeoff for future work. That is, we seek to characterize the probability of an undercollateralization event *given that the agent is selecting the profit-maximizing* $R^*(M,S)$.

Let $F : [0,\infty] \to [0,1]$ denote the cumulative distribution function of the price-shock distribution D. Then the probability of undercollateralization in the presence of a profit-maximizing agent, denoted p_u, is given by

$$p_u(M,S) \triangleq F\left(\frac{S}{R^*(M,S)}\right). \tag{11}$$

That is, undercollateralization occurs when d is realized *below* the threshold S/R, as in the first condition of (9); equation (11) is simply the cdf evaluated at this point with $R = R^*(M,S)$[2].

[2]This threshold is triggered since all collateral is exhausted in paying the price-stable token debt *after* the payment of the margin-call penalty S.

III. Our Contributions

The central goal of this paper is to characterize how the system designer's choice of M and S affect the emergent probability of undercollateralization p_u. As a first step, in this paper we perform a numerical study of the undercollateralization risk in the presence of a price disturbance that is distributed lognormally. That is, we consider a situation in which the price disturbance d is given by the formula

$$d = \exp(s), \tag{12}$$

where s is drawn from normal distribution $\mathcal{N}(\mu, \sigma^2)$ with mean μ and variance σ^2. Under this distribution, if $\mu = 0$ we have the convenient fact for all $\delta > 0$ that

$$f(\delta) = f(1/\delta). \tag{13}$$

That is, this distribution models symmetric *multiplicative* price shocks: when $\mu = 0$, the price is equally likely to double as it is to halve.

Mathematically, given μ and σ as above, the probability density function of this distribution is given by

$$f(x) = \frac{1}{\sigma x \sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\log(x-\mu)}{s}\right)^2\right). \tag{14}$$

A. The interplay between MCR and MSSR

First, we note that regardless of the distribution D from which price shocks are drawn, the system operator's choice of M affects agent profits *only* if $S > 1$. That is, if the operator selects $S = 1$, this effectively curtails their ability to have a nuanced effect on agent behavior.

Proposition III.1. *If $S = 1$, then for any fixed R, the agent's expected profit $P(R; M, S)$ is a constant function of M.*

Proof. Consider the *ex post* profit function given by (9), and let $S = 1$. Then the function collapses to

$$\begin{aligned} P(R, d; M, 1) &= \begin{cases} \frac{1}{R} - 1 & \text{if } d \in [0, 1/R) \\ \frac{1}{R}\left(1 - \frac{1}{d}\right) & \text{if } d \in [1/R, M/R) \\ \frac{1}{R}\left(1 - \frac{1}{d}\right) & \text{if } d \in [M/R, \infty) \end{cases} \\ &= \begin{cases} \frac{1}{R} - 1 & \text{if } d \in [0, 1/R) \\ \frac{1}{R}\left(1 - \frac{1}{d}\right) & \text{if } d \in [1/R, \infty), \end{cases} \end{aligned} \tag{15}$$

which is clearly a constant function of M. Since the *ex post* profit is constant in M, then so must the expected profit be as well. □

Thus, it may be helpful to think of S as a "switch" which activates M. If S is not used (i.e., no penalty is charged for margin calls), then M loses its effectiveness to influence agent behavior. This suggests that the system operator should always maintain $S > 1$.

From here, let us assume that $S > 1$. Our next result considers the effect of M on agent profits. Intuitively, one would expect that increasing M would strictly decrease the agent's expected profit. Proposition III.1 demonstrates that this is not generally true (in particular, it fails when

$S = 1$). However, our next result shows that for a wide range of price shock distributions, the agent's profit (for fixed R) is strictly decreasing in M.

Proposition III.2. *Let $S > 1$, let $R > M$, and let price shock distribution D be such that its pdf has $f(M/R) > 0$. Then the agent's expected profit is strictly decreasing in M:*

$$\frac{\partial}{\partial M} P(R; M, S) = \frac{f(M/R)}{MR} \times (1 - S) < 0. \tag{16}$$

Proof. Let $f(t)$ be the pdf of distribution D. Then when the agent selects collateral ratio R, the expected profit ratio can be computed from (9) as

$$P(R; M, S) \triangleq \mathop{\mathbb{E}}_{d \sim D} [P(R, d; M, S)] = \frac{1}{R} - \int_0^{\frac{S}{R}} f(t) dt - \frac{S}{R} \int_{\frac{S}{R}}^{\frac{M}{R}} \frac{f(t)}{t} dt - \frac{1}{R} \int_{\frac{M}{R}}^{\infty} \frac{f(t)}{t} dt. \tag{17}$$

The partial derivative of $P(R; M, S)$ can be computed from (17) as

$$\frac{\partial}{\partial M} P(R; M, S) = -\frac{S f(M/R)}{MR} + \frac{f(M/R)}{MR} \tag{18}$$

which completes the proof of Proposition III.2. □

B. Optimal agent behavior as a function of MCR

The complexities of the system render a full theoretical analysis challenging; accordingly, we perform a simple numerical simulation study on the effect of the MCR parameter M on the risk of undercollateralization events. To demonstrate how this parameter affects p_u, we first plot the agent's expected profit ratio $P(R; M, S)$ as a function of collateral ratio R for a fixed distribution D (with lognormal parameters $\mu = 0.033$ and $\sigma = 0.2$, fixed $S = 1.01$, and varying M. See Figure 1 for a depiction of this.

Remark 1. *The simulations depicted in Figure 1 demonstrate that the agent's optimal choice of collateral ratio can be extremely sensitive to the system operator's choice of M. In particular, at a threshold of approximately $M \approx 1.53$, $P(R; M, S)$ is discontinuous in M, increasing suddenly from about 1.53 to about 1.85.*

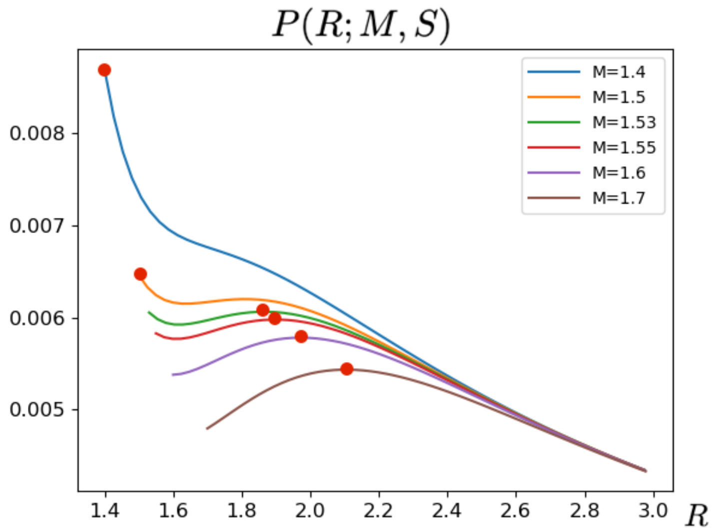

Figure 1: Plots of agent's expected profit ratio $P(R; M, S)$ as a function of R for fixed lognormal distribution parameters of $\mu = 0.033$ and $\sigma = 0.2$, and $S = 1.01$ and various values of M. Note that when M is low, e.g. $M = 1.4$, the agent's optimal decision (that is, the maximizer of the corresponding trace on the chart, marked approximately by the red discs) is $R^*(M, S) = M$. However, increasing M has the tendency to "pull down" the left-hand end of the trace. When $M \approx 1.5$, this gives rise to a local maximum in the profit function away from $R = M$; around $M \approx 1.53$, this local maximum becomes the global maximum and the agent's optimal collateral ratio "snaps" to the right, yielding $R^*(M, S) > M$.

C. Undercollateralization risk as a function of MCR

To understand how these agent decisions impact $p_u(M, S)$, we then compute the optimal collateral ratio $R^*(M, S)$ as a function of M for several values of S, and report these results in Figure 2. We find several features of Figure 2 worthy of note.

Remark 2. *The probability of an undercollateralization event, p_u, can be sharply (discontinuously) dependent on M, but the presence of this discontinuity depends on the value of S. Considering the right-hand plot in Figure 2, note that around $M \approx 1.53$, there is a discontinuity in the p_u plot corresponding to $S = 1.01$. This indicates that p_u is an extremely sensitive function of the incentive parameters M and S in complex ways that are not* a priori *obvious and are deserving of further study.*

Remark 3. *The probability of undercollateralization, p_u, is not monotone in S, and the nature of its non-monotonicity is* not consistent over all values of M. *That is, consider $M' = 1.5$; here, we have that p_u is locally increasing in S: $p_u(M', 1.01) > p_u(M', 1.005)$. Alternatively, consider $M'' = 1.6$; here, we have that p_u is locally* decreasing *in S: $p_u(M', 1.01) < p_u(M', 1.005)$. This indicates that without careful analysis, it may be nearly impossible to determine how to select incentive parameters*

Remark 4. *For low-enough S and very low M, the optimal collateral ratio appears to be $R^*(M, S) = M$. Consider the left-hand plot of Figure 2: here, for $S \in \{1.005, 1.01\}$, the agent's optimal value of R is* *the same. That is, in this regime, S has no effect on collateral ratios. However, in the right-hand plot of Figure 2, in that same regime, it is evident that the probability of undercollateralization actually* is *a function of S, and that higher S leads to higher p_u. That is, in this regime, this suggests that it is strictly better to set $S = 1$.*

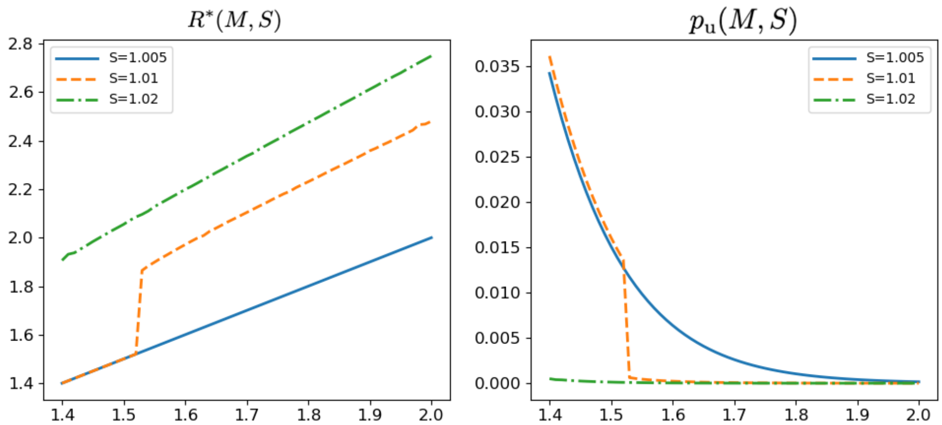

Figure 2: Plots of system behavior as a function of M for fixed lognormal distribution parameters of $\mu = 0.033$ and $\sigma = 0.2$ and various values of S.
Left: Agent's optimal collateral ratio $R^*(M, S)$ with respect to M. Note that when S is low, the agent's optimal action is to set $R = M$, but that larger values of S render other, less-risky collateral ratios optimal.
Right: Probability of undercollateralization $p_u(M, S)$with respect to M. Several features are of note here: first, when $S = 1.02$, the probability of undercollateralization is extremely low, despite the fact that only a 2% penalty is assessed the agent in the event of a margin call. Second, when $S \leq 1.01$, the probability of undercollateralization (p_u) is sharply dependent on the value of M, can be as high in these simulations as 3.5%, and for $S = 1.01$, p_u is discontinuous around $M = 1.53$. That is, the probability of undercollateralization is *extremely* sensitive to the operator's selection of M.

IV. Discussion and Future Work

Our analytical results in Propositions III.1 and III.2 suggest that selecting $S > 1$ is a crucial element of an effective incentive design; any value of $S > 1$ ensures that M is an effective tool for influencing the behavior of agents.

Taken together, Remarks 1, 2, 3 and 4 illustrate that even on this carefully simplified model of the BitShares incentive system, emergent behavior among agents can be extremely complex and unintuitive. If behavior is challenging on a simple model, we expect that it is likely to be more challenging as the model's complexity increases to match the real-world functions of the system. In particular, we recommend that careful attention be paid to the following key things:

- The behavior of token holders may be *extremely* sensitive to small changes in MCR. This may be exploited to increase liquidity (i.e., by decreasing MCR slightly to incentivize the creation of additional price-stable tokens), but it may be very difficult to predict its precise effects and can easily increase the overall risk in the system.

- The effects of MSSR on token holder behavior may be unintuitive, and essential aspects of their character may be highly dependent on MCR. In our simulations, we found that for low MCR, it is strictly better to set MSSR very close to 1 rather than at a moderate value such as 1.01, as noted in Remark 4. However, for a somewhat higher MCR, as we note in Remark 3, this situation is reversed and risks decrease with S.

A. Future Work

Clearly, the most important priority is to adapt the present model to a multiagent dynamic context which more accurately captures the complexities of the agents' decisions. Modeling this as a one-shot decision process has benefits in its simplicity, but clearly misses some important aspects of the real-world system. In particular, in the real BitShares system, agents have the ability to update their collateral ratios over time as the core token price evolves, and integrating this into our model will be crucial to generate high-fidelity predictions.

Furthermore, there are intrinsic game-theoretic aspects to this system, due to the market dynamics and the fact that in the real system, not all agents are charged the margin-call penalty every time. Lastly, an important aspect that is missed in our analysis is that in the actual BitShares system, there is a much more nuanced relationship between the external market and the actions of the agents committing collateral to create price-stable tokens. That is, the market value of the price-stable tokens can actually fluctuate somewhat, and this fluctuation likely dramatically impacts the incentives faced by core token holders.

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