Fundamental Scalability Laws for Wireless Networks

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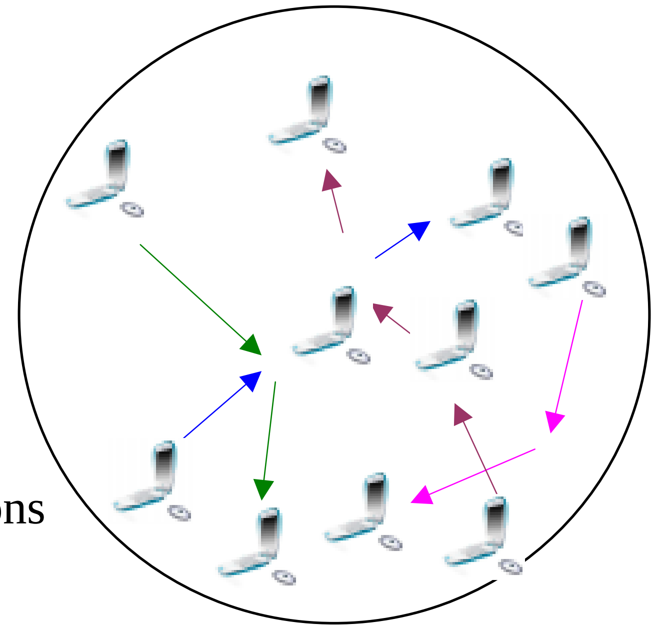
Model of Wireless Networks

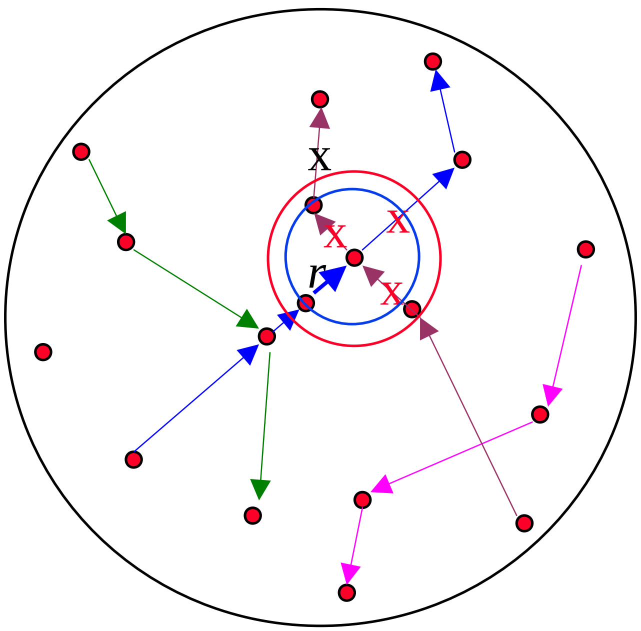
- *n* nodes located in a disk of area *A* sq.m
- Each can transmit at *W* bits/sec
- Shared wireless channel: Interference can occur between transmissions
  - Two models for successful reception of transmissions
    - Protocol Model
    - Physical Model (Signal-to-Interference Ratio)



- Question: How does the performance scale with the number of nodes *n*?
- Two metrics examined
  - Throughput for each node: Measured in Bits/Sec
  - Traffic carrying capacity of entire network: Measured in Bit-Meters/Sec

Protocol and Physical Models

- Protocol Model
  - Transmission at a range r is successful if there are no other transmitters within a distance $(1+\Delta)r$ of the receiver*



*Or interference caused within $(1+\Delta)r$ of a transmitter

- Physical Model (SIR Model)

$$\text{SIR Ratio} = \frac{P_i r_i^{-\alpha}}{N + \sum_{j \neq i} P_j r_j^{-\alpha}} \geq \beta$$

  - P_i = power of i-th node
  - N = Noise power
  - r_j = Distance of j-th transmitter from given receiver
  - $r^{-\alpha}$: Signal Power Path Loss, $\alpha > 2$
  - β = SIR for successful reception

Best Possible Scenario

- Protocol Model: Network can transport $\Theta\left(W\sqrt{An}\right)$ bit-meters/sec

$$\frac{W}{1+2\Delta}\frac{n}{\sqrt{n}+\sqrt{8\pi}} \le \text{Best case capacity for Protocol Model} \le \sqrt{\frac{8}{\pi}}\frac{W}{\Delta}\sqrt{n} \quad \text{bit-meters/sec}$$

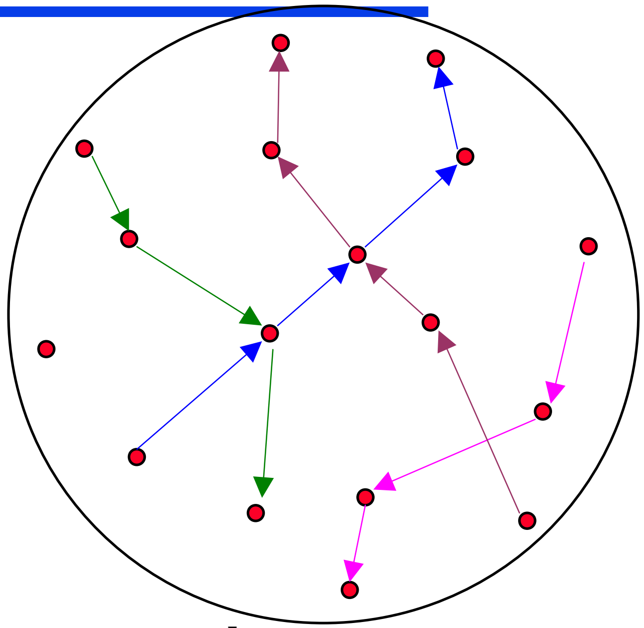
$$\frac{1}{\left(16\beta\left(2^{\alpha/2}+\frac{6^{\alpha-2}}{\alpha-2}\right)\right)^{1/\alpha}}\frac{Wn}{\sqrt{n}+\sqrt{8\pi}} \le \text{Best case capacity for Physical Model bit-meters/sec} \le \left(\frac{2\beta+2}{\beta}\right)^{1/\alpha}\frac{W}{\sqrt{\pi}}n^{(\alpha-1)/\alpha}$$

Can be sharpened to $O\left(\sqrt{n}\right)$ if $P_{max}/P_{min} < \beta$

- Optimal network
  - » Optimally located nodes, destinations, demands for OD-pairs
  - » Optimal spatial and temporal scheduling, routes, ranges for each transmission
- If equitably divided each node can send $\Theta\left(W\sqrt{\frac{A}{n}}\right)$ bit-meters/sec

Random Network Scenario

- Protocol Model: n nodes randomly located in disk of unit area
  - Each node chooses random destination
  - Equal throughput λ bits/sec for all OD pairs
  - Each node chooses same range r
- Each node can send $\Theta\left(\frac{1}{\sqrt{n \log n}}\right)$ bits/sec even with
  - With best choice of spatio-temporal scheduling, ranges and routes



- Definition of capacity

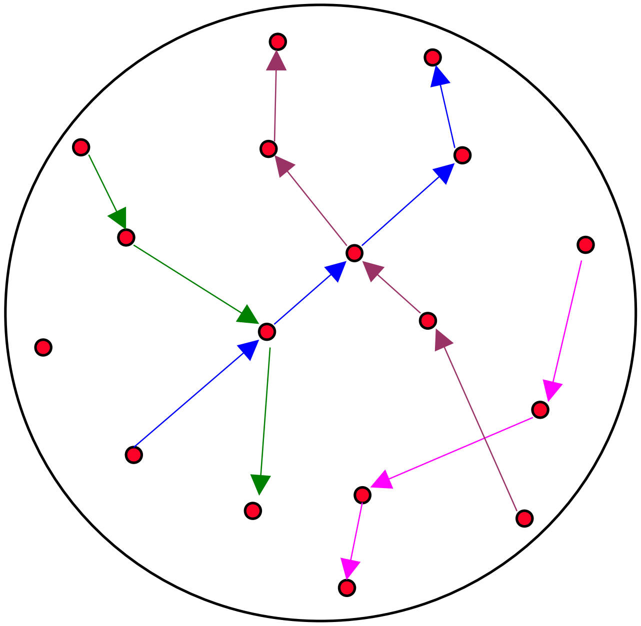
$$\lim_{n\to\infty} \Pr\left(\lambda(n) = \frac{c}{\sqrt{n \log n}} \text{ is feasible}\right) = 1, \text{ and}$$

$$\lim_{n\to\infty} \mathrm{P}\left(\lambda(n) = \frac{c'}{\sqrt{n \log n}} \text{ is feasible}\right) = 0$$

Sharp cutoff phenomenon

Physical Model: Random Network

- n nodes randomly located in disk of unit area
  - Each node chooses random destination
  - Equal throughput λ bits/sec for all OD pairs
  - Each node chooses same power level P



- Theorem

$$\Theta\left(\frac{1}{\sqrt{n\log n}}\right) \leq \quad \lambda(n) \quad \leq \Theta\left(\frac{1}{\sqrt{n}}\right) \text{ bits/sec}$$

- With best choice of routes, hops, spatio-temporal scheduling

Implications for designers

- Design networks with few nodes, or scaled down bandwidth, or support mainly nearest neighbor communications
- Splitting into several sub-channels (TDMA, FDMA, CDMA) does not help in increasing capacity
- Power consumption: Busy fraction of modems is $\Theta\left(\frac{1}{\log n}\right)$
- Range of transmissions: Scaled length of hops is $\Theta\left(\sqrt{\frac{\log n}{n}}\right)$
- Architecture for providing optimal capacity
  Group nodes into cells of size O(log *n*) - one node in each cell serving as relay
- *kn* randomly placed relay nodes increase capacity by factor $\sqrt{k}$
- Directed transmissions will help